New constraints for QCD matter from improved Bayesian parameter estimation in heavy-ion collisions at LHC

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Based on PRC 104 (2021) 054904, arXiv:2111.08145

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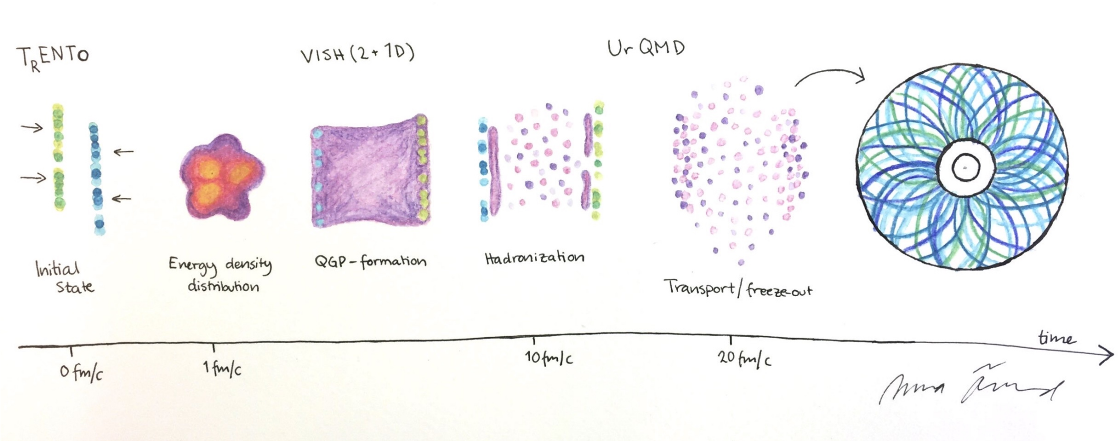
Friday 1st July, 2022

Future Heavy-Ion Collision Projects: from LHC to FCC, July 1-2, 2022
Yeosu, South Korea

Yeosu, 2015, D. J. Kim

UNIVERSITY OF JYVÄSKYLÄ CERN CoE QM HELSINKI INSTITUTE OF PHYSICS

The different stages of Heavy-Ion collisions

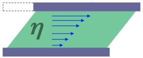


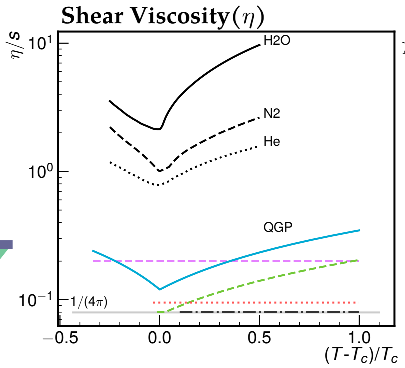
Credits to Anna Onnerstad

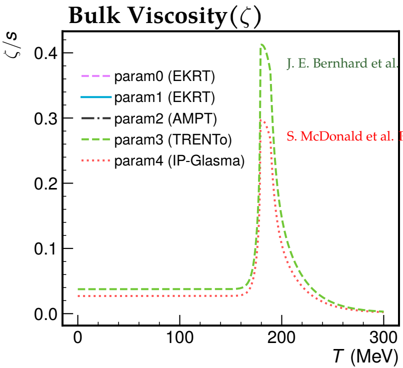
$$T^{\mu\nu} = e u^{\mu} u^{\nu} - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \qquad \delta_{\mu} T^{\mu\nu} = 0$$

based on https://github.com/Duke-QCD

Transport properties in Heavy-ion collisions

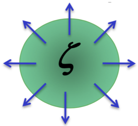






J. E. Bernhard et al. PRC 94 024907 (2016)

S. McDonald et al. PRC 95, 064913 (2017)



ALICE, JHEP05 (2020) 085

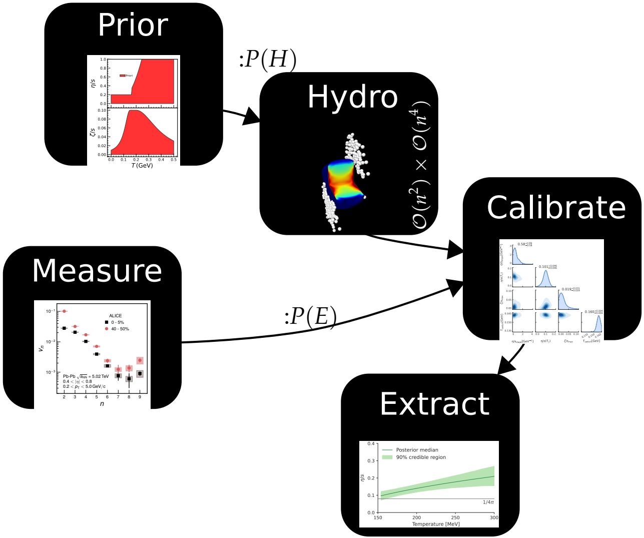
$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\text{slope}}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\text{curve}}}, (\zeta/s)(T) = \frac{(\zeta/s)_{\max}}{1 + \left(\frac{T - (\zeta/s)_{T_{\text{peak}}}}{(\zeta/s)_{\text{width}}}\right)^2}$$

Latest MAP parameters, Jyvaskyla (2022)

Table: Input parameter ranges for the initial condition and hydrodynamic models.

Parameter	Description	Range	MAP
N(2.76 TeV)	Overall normalization (2.76 TeV)	[11.152, 18.960]	14.373
N(5.02 TeV)	Overall normalization (5.02 TeV)	[16.542, 25]	21.044
p	Entropy deposition parameter	[0.0042 , 0.0098]	0.0056
σ_k	Std. dev. of nucleon multiplicity fluctuations	[0.5518, 1.2852]	1.0468
d^3_{min}	Minimum volume per nucleon	$[0.889^3, 1.524^3]$	1.2367^3
τ_{fs}	Free-streaming time	[0.03, 1.5]	0.71
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$	[0.135, 0.165]	0.141
$\eta/s(T_c)$	Minimum $\eta/s(T)$	[0, 0.2]	0.093
$(\eta/s)_{\mathrm{slope}}$	Slope of $\eta/s(T)$ above T_c	[0, 4]	0.8024
$(\eta/s)_{\mathrm{curve}}$	Curvature of $\eta/s(T)$ above T_c	[−1.3, 1]	0.1568
$(\zeta/s)_{\mathrm{peak}}$	Temperature of $\zeta/s(T)$ maximum	[0.15, 0.2]	0.1889
$(\zeta/s)_{\mathrm{max}}$	Maximum $\zeta/s(T)$	[0, 0.1]	0.01844
$(\zeta/s)_{\mathrm{width}}$	Width of $\zeta/s(T)$ peak	[0, 0.1]	0.04252
T_{switch}	Switching / particlization temperature	[0.135, 0.165]	0.1595

Bayesian parameter estimation



Bayes' theorem:

$$\boxed{P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}}$$

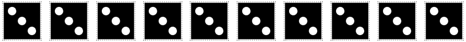
$$, P(E) = \sum_{i=1}^{n} P(E|H_i) P(H_i)$$

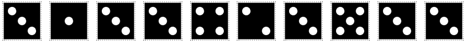
- Find optimal set of model parameters that best reproduce the experimental data
- Utilize constraints, such as flow observables, to help narrow down the $\eta/s(T)$ and such.

Testing a single set of parameters requires $\mathcal{O}(10^4)$ hydro events, and evaluating eight different parameters five times each requires $5^8 \times 10^4 \approx 10^9$ hydro events.
That's roughly 10^5 CPU years!

Bayesian Theorem - a simple quiz?

$$\boxed{P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}}$$

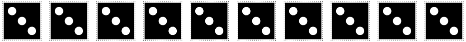
 1/6 1/6......1/6, Prob(3) for 11th? It is 1/6

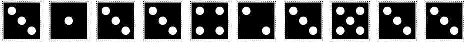
 Up to 10th Prob(3) was 6/10, Prob for 11th?

Probablity to have  **for 11th throw?**

Bayesian Theorem

$$\boxed{P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}}$$

 1/6 1/6......1/6, Prob(3) for 11th? It is 1/6

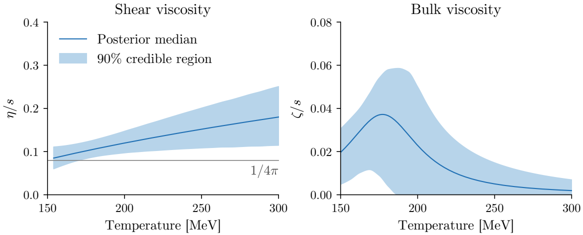
 Up to 10th Prob(3) was 6/10, Prob for 11th?

Probablity to have  **for 11th throw is** $\frac{6+1}{10+1}$

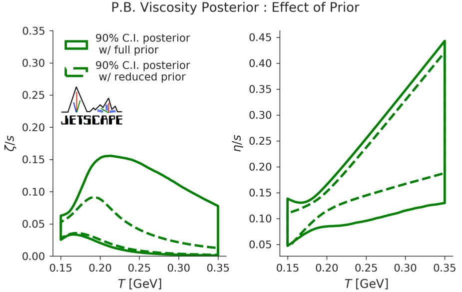
Bayesian parameter estimation: Previous work

JETSCAPE T$_\mathrm{R}$ENTo+MUSIC+SMASH

Duke T$_\mathrm{R}$ENTo+VISH(2+1D)+UrQMD



Steffen A. Bass *et. al*, Nature Physics (2019)

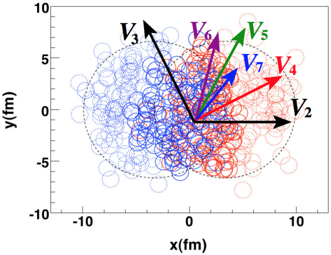


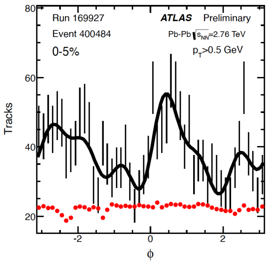
JETSCAPE Collaboration, PRC **103** (2021) 054904

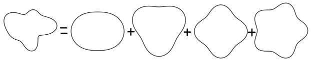
- Low to moderate temperature dependence on $\eta/s(T)$
- Moderate magnitude of $\zeta/s(T)$ ($\sim 0.1\times$ w.r.t lattice QCD(PRL. **94**, 072305 (2005)))
- Large uncertainty for both $\eta/s(T)$ and $\zeta/s(T)$.
- Subsequent studies with still limited observables:
  - J. Auvinen *et al.* PRC. **102**, 044911 (2020)
  - G. Nijs *et al.* PRL. **126**, 202301 (2021)

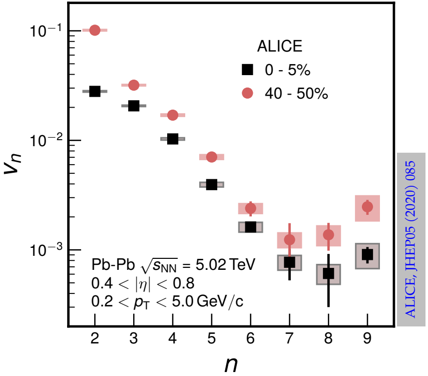
Uncertainties need to and can be further improved.
Only low-order harmonic v_n was used, including a *limited set of mostly 2.76 TeV* observables.

Higher Flow Harmonics and flow fluctuation







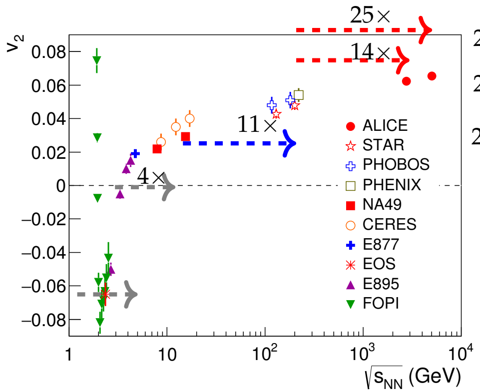


$$P(\varphi) \propto \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\varphi}$$

$$V_n \equiv v_n\{\psi_n\} e^{in(\psi_n - \phi)}$$

- Sensitive to initial state geometry and properties of the expanding QGP (viscosity(η/s), equation of state)

v_2 VS $\sqrt{s_{NN}}$ AND FLOW POWER SPECTRUM



2015 LHC 5.02TeV CERN

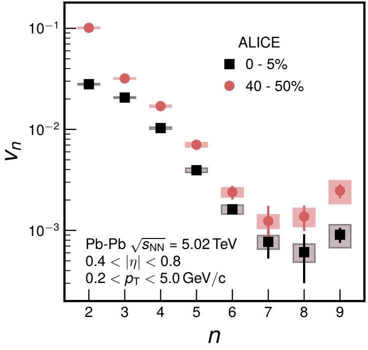
2010 LHC 2.76TeV CERN

2000 RHIC 200GeV USA

90s SPS 17GeV CERN

80s AGS 4GeV USA

2020, cerncourier[Going with the flow]



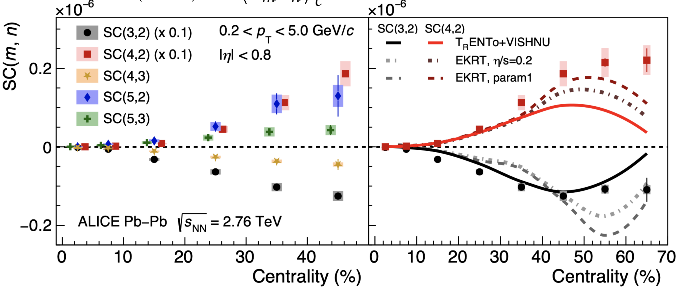
ALICE, Phys. Rev. Lett. 105 (2010) 252302

ALICE, JHEP05 (2020) 085

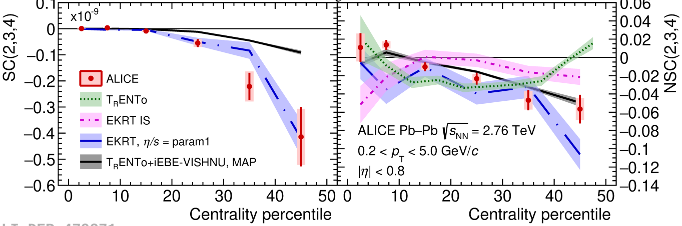
Measured the largest flow v_2 in 2010!
Measured the largest harmonic order flow (up to v_9) so far, 2020

High precision flow results and New developments- Symmetric Cumulants

$$\mathrm{SC}(m,n) = \langle v_m^2 v_n^2 \rangle_c$$



$$\mathrm{SC}(k,l,m) = \langle v_k^2 v_l^2 v_m^2 \rangle_c$$



ALI-DER-479271

ALICE, Phys. Rev. Lett. 117 (2016) 182301

ALICE, Phys. Rev. C 97 no. 2, (2018) 024906

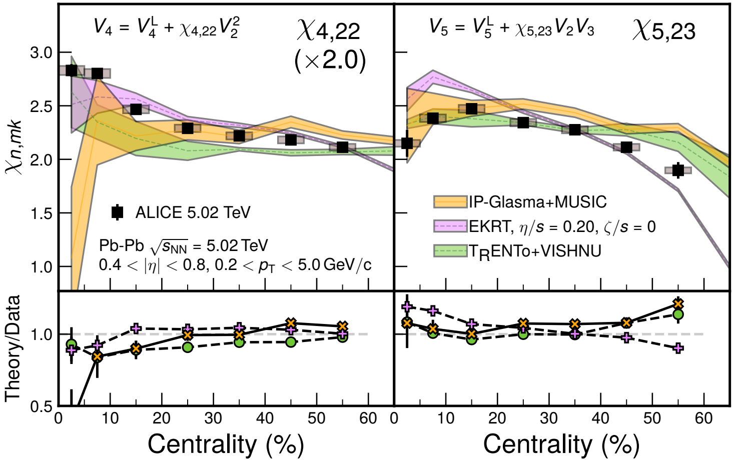
- Accessing the temperature dependence of $\eta/s(T)$

ALICE, Phys. Rev. Lett. 127 (2021) 092302

- $\eta/s(T)$ and accessing $\zeta/s(T)$

- Very challenging measurements because of their required high precisions (i.e 10^{-6} $\mathrm{SC}(m,n)$, 10^{-12} for $\mathrm{SC}(k,l,m)$) and difficulties in correcting experimental biases.
- Symmetric Cumulants(Standard Candle)

Improving results with higher harmonics and more precision - Non-linear flow modes



- Higher order v_n's (n>3) were studied → non-linear dependence on lower orders
- Characterised by the non-linear flow mode coefficients, $\chi_{n,mk}$
- Better sensitivity to $\eta/s(T)$.

ALICE, JHEP05 **085** (2020)

Our arsenal of observables - stochastic approach

- Together various flow observables cover the sensitivity for all components of transport properties.

Name	Symbol	Measure	Sensitivity-stochastic approach
Flow coefficients	v_n	System expansion and anisotropy of the flow	Average $\langle \eta/s \rangle$ and $\zeta/s(T)$ peak
(Normalized) Symmetric cumulants	(N)SC(k, l, m)	Correlation between magnitudes of flow harmonics	$\eta/s(T)$ temperature dependence
Linear and non-linear contributions	$v_{n,\mathrm{L}}$, $v_{n,mk}$	Magnitude of the linear and non-linear contributions	$\eta/s(T)$ and initial conditions, not used
Non-linear flow mode coefficients	$\chi_{n,mk}$	Quantification of the non-linear response	$\eta/s(T)$ at the freeze-out
Symmetry-plane correlations	$\rho_{n,mk}$	Correlations between the directions of flow harmonics	$\eta/s(T)$

Thanks to excellent ALICE papers over years:

- Phys.Rev.Lett. 117 (2016) 182301, Phys.Lett. B773 (2017) 68, Phys.Rev. C 97 (2018) 024906, JHEP05 (2020) 085, Phys.Lett. B818 (2021) 136354, Phys.Rev.Lett. 127 (2021) 092302 - flow
- Phys.Rev.Lett. 106 (2011) 032301, Phys.Rev.C 88 (2013) 044910, Phys.Lett. B772 (2017) 567-577, Phys.Rev.C 101, 044907 (2020) - N_{ch} and $\langle p_{\mathrm{T}} \rangle$

Our arsenal of observables

Duke (2019)

2.76 TeV

- PID[1] mult. and N_{ch}
- Transverse energy E_{T}
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- $\delta p_{\mathrm{T}} / \langle p_{\mathrm{T}} \rangle$
- v_2 to v_4

5.02 TeV

- N_{ch}
- v_2 to v_4

[1] Jyvaskyla (2021)

5.02 TeV

- PID[2] mult. and N_{ch}
- v_2 to v_7
- NSC(3,2) to NSC(4,3)
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- $\chi_{4,22}$ to $\chi_{6,mk}$

[2] Jyvaskyla (2022)

2.76 TeV

- N_{ch}
- NSC(3,2), NSC(4,2)
- NSC(2,3,4), NSC(2,3,5)
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- v_2 to v_4
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$

5.02 TeV

- PID[2] mult. and N_{ch}
- NSC(3,2) to NSC(4,3)
- PID $\langle p_{\mathrm{T}} \rangle$
- v_2 to v_7
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$

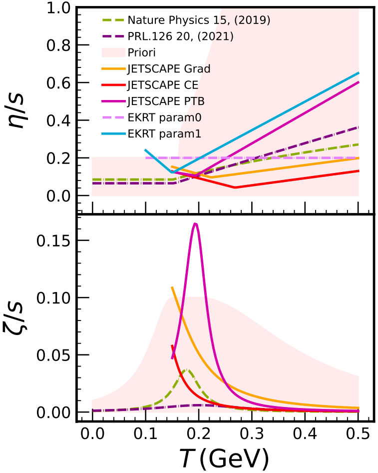
All reference data based on ALICE measurements.

Red: Missing from other group(Duke etc)
Blue: New since our PRC.
Orange: Not used in our studies.

[1] $\pi^{\pm}$, $K^{\pm}$ and $p^{\pm}$
[2] $p^{\pm}$

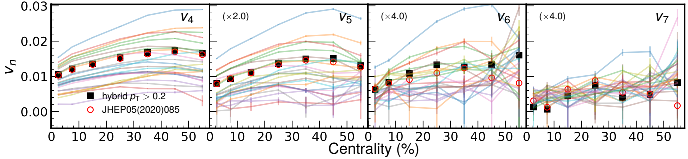
[1]. J.E. Parkkila, A. Onnerstad, D.J. Kim, PRC **104** (2021) 054904
[2]. J.E. Parkkila, A. Onnerstad, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv:2111.08145

Analysis steps and priori

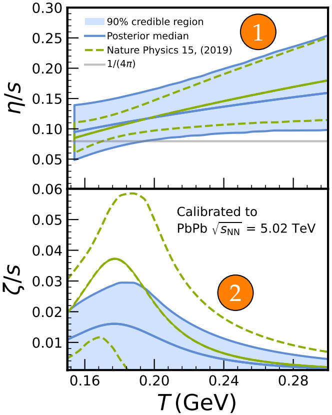


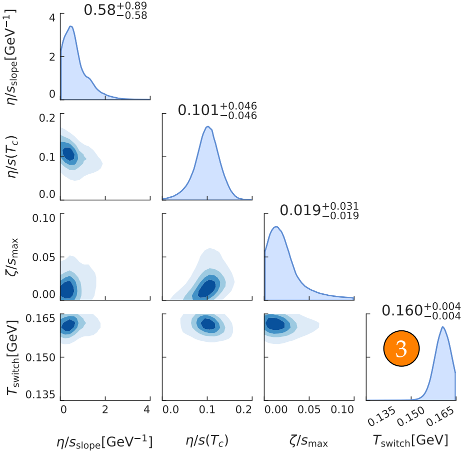
PRC 104 (2021) 054904

1. Choose prior parameter range based on results from 2019
2. Run hydro `TRENTo+VISH(2+1D)+UrQMD` for 500 parameterizations, 3-5 million events ($\times$100 previous).
3. Calculate observables using our experimental framework
4. Train emulator and setup/run Bayesian analysis



Results: Jyvaskyla (2021) (5.02 TeV only) – higher statistics run

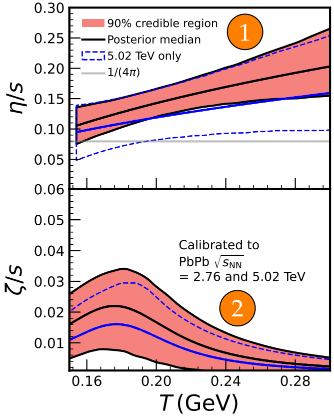


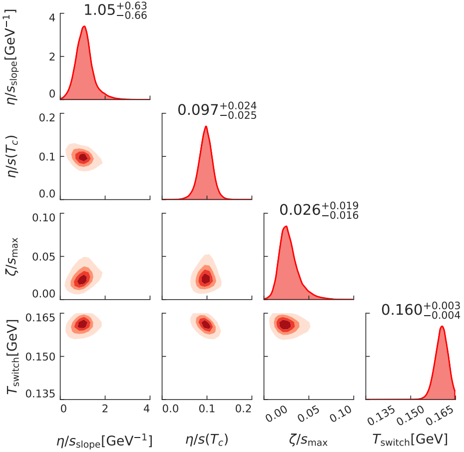


1. Similar $\eta/s(T)$ to Duke (2019)
2. Lower $\zeta/s(T)$ – much lower to previous calculations
3. Higher switching temperature T_{switch} (vs. Duke 152 MeV)

- Additional observables have reduced $\zeta/s(T)$.
- However, one collision energy only limits the potential of the additions.

Result: Jyvaskyla (2022) – combined collision energy analysis (2.76 + 5.02 TeV)

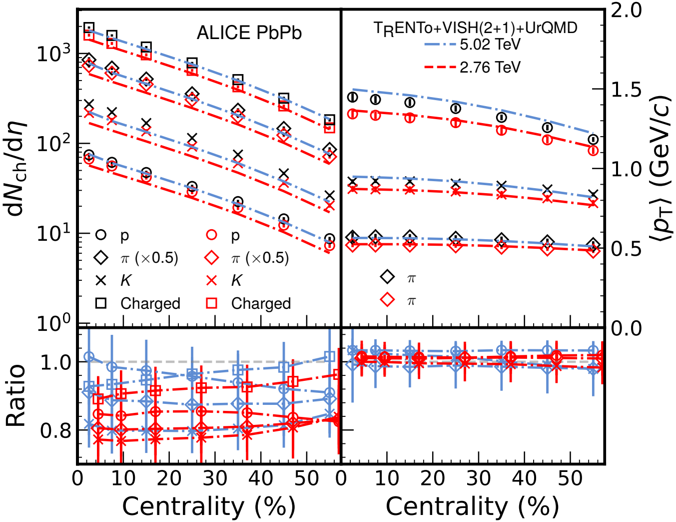




1. Significantly improved $\eta/s(T)$ uncertainty
2. Non-zero $\zeta/s(T)$
3. Overall better convergence for parameter components

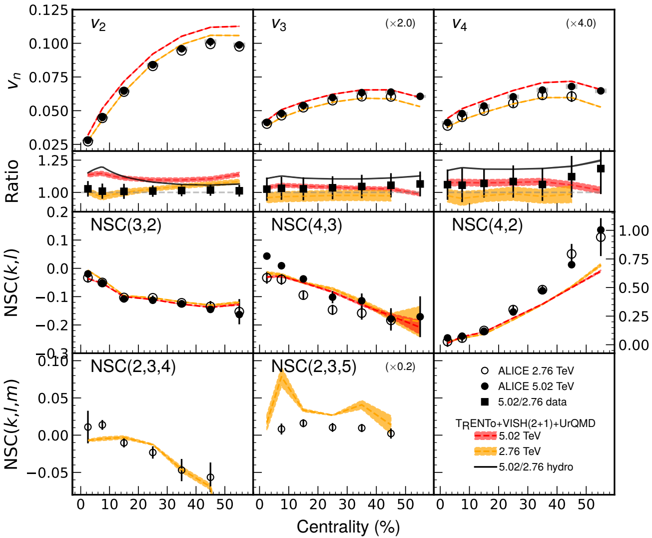
- Together with two collision energies and added observables, the uncertainty has reduced!

PID multiplicity and $\langle p_T \rangle$



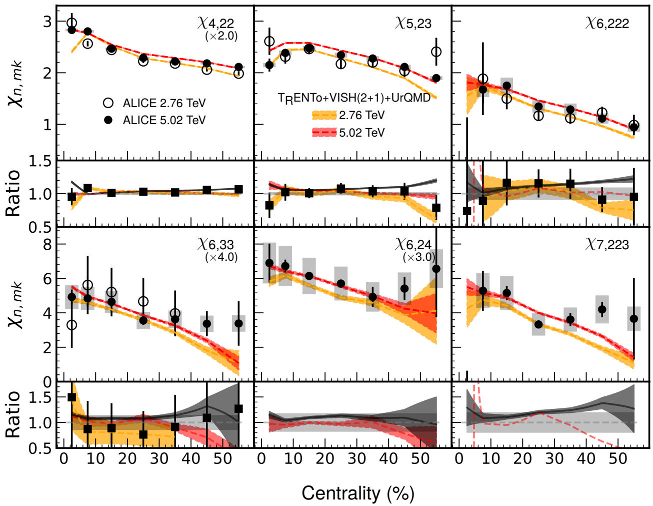
- Agreement for charged particle yield in 2.76 TeV and 5.02 TeV
- 10–20% difference for PID multiplicity
- Qualitative agreement for $\langle p_T \rangle_{\pi,K}$

v_n and Symmetric Cumulants



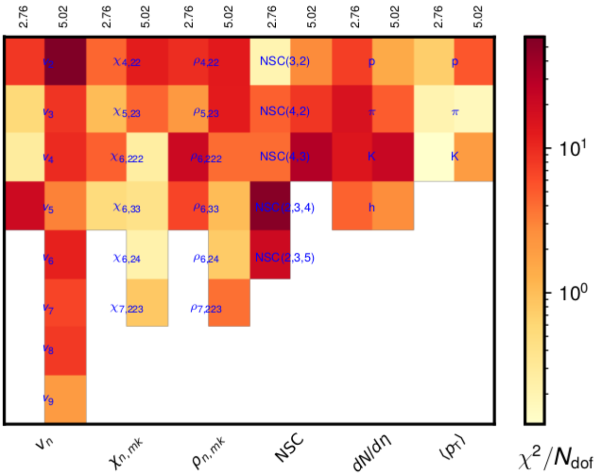
- Good agreement for 2.76 TeV v_n, overestimated v_2 for 5.02 TeV by $\sim 10\%$
- Magnitude and centrality dependence of NSC well captured. Further improved estimate for NSC(4,2).
- Good agreement for NSC(2,3,4). NSC(2,3,5) overestimated.

Non-linear flow coefficients



- Qualitative agreement in both beam energies for all mode coupling coefficients.
- See arXiv:2111.08145 for all graphs.

Remaining Concerns?



- Higher energy description worse for all observables except for:
  - v_5
  - $\chi_{6,222}$
  - charged particle multiplicity
- Concerns
  - overestimated v_n for 5.02 TeV by $\sim 10\%$
  - still underestimated NSC(4,2)
  - overestimated NSC(2,3,5)
  - PID multiplicity (especially $\pi^{\pm}$)
- Why?
  - Reduction of the uncertainties is understood?

Sensitivity of the observables to parameters

Sensitivity of the observables: $S[x_j] = \Delta/\delta.$, where $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$.

	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	$\chi_{4,22}$	$\chi_{5,23}$	$\chi_{6,222}$	$\chi_{6,33}$	$\rho_{4,22}$	$\rho_{5,23}$	$\rho_{6,222}$	$\rho_{6,33}$	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)	NSC(2, 3, 4)*	NSC(2, 3, 5)*	NSC(2, 4, 6)*
τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
$(\eta/s)_{slope}$	0.00	0.14	0.19	0.19	0.18	0.49	1.43	2.10	3.30	0.26	0.13	0.81	0.85	1.74	1.37	3.03	1.36	0.70	7.02	1.70	34	30	1.06
T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
$\eta/s(T_c)$	0.00	0.08	0.22	0.21	0.14	1.65	3.29	4.87	6.00	0.95	3.84	2.92	5.44	2.53	1.27	2.85	2.76	1.61	15	9.80	22	29	14
$(\eta/s)_{crv}$	0.00	0.17	0.05	0.05	0.05	0.19	0.38	0.53	0.69	0.20	0.36	0.55	0.69	0.40	0.40	0.73	0.75	0.13	1.81	4.72	2.20	8.41	6.17
$(\zeta/s)_{peak}$	0.01	0.44	1.16	1.01	0.73	2.26	2.67	3.79	6.40	0.59	0.32	1.48	1.11	0.34	1.59	1.04	1.15	1.49	1.49	2.57	3.74	2.04	15
$(\zeta/s)_{max}$	0.00	0.07	1.05	0.96	0.77	0.37	0.13	0.12	0.37	0.20	0.01	0.53	1.12	0.52	0.64	1.26	0.59	0.44	1.43	8.00	3.02	1.58	2.91
$(\zeta/s)_{width}$	0.00	0.01	0.20	0.15	0.07	0.19	0.04	0.09	0.33	0.09	0.26	0.40	1.69	0.39	1.66	1.55	1.35	0.38	1.41	1.21	1.77	23	19
T_{switch}	0.01	1.34	0.25	0.21	0.16	1.30	3.51	4.64	5.86	0.77	0.45	2.03	3.34	3.95	2.42	1.98	1.00	1.96	17	13	4.08	0.47	6.22

$S[x_j]$ (colour scale: 2, 4, 6, 8)

- NSC(m,n) and NSC(k,l,m) are among the most sensitive observables followed by v_n and $\chi_{n,mk}$.
- The precision measurements of observables, reflecting mostly non-linear responses, are crucial.

Summary

Success:

- Higher harmonic orders and non-linear flow observables $\rightarrow$ better constraints.
- Improved the overall uncertainty by $\times$ 2 by combining two beam energy data.
- As a bonus, sensitivities of the observables are now quantified
  $\rightarrow$ precision measurements of observables, reflecting non-linear hydrodynamic responses.

Challenges:

- 10% difference for v_2 (5.02 TeV)
- NSC description improved except for NSC(4,2)
- Remaining discrepancy for PID multiplicity (especially $\pi^{\pm}$)
- Improving the initial state model, with dynamical collision model or subnucleon structure à la IP-Glasma, might help us to improve the results.

Outlook

Experiments

- RHIC data (AuAu collisions) - Energy and system size dependence
- LHC pPb and pp data - System size dependence
- Use new observables
  - Higher order ($n > 5$) Symmetric cumulants
  - Improved Symmetric Plane Correlation (SPC) : independent from flow magnitude correlations
  - Asymmetric Cumulants (AC)

Theory

- Improving the initial conditions with
  - EKRT
  - IP+Glasma
- Testing hydro limit of small systems?
- Role of the small system.

Thanks

Thank you for your attention!

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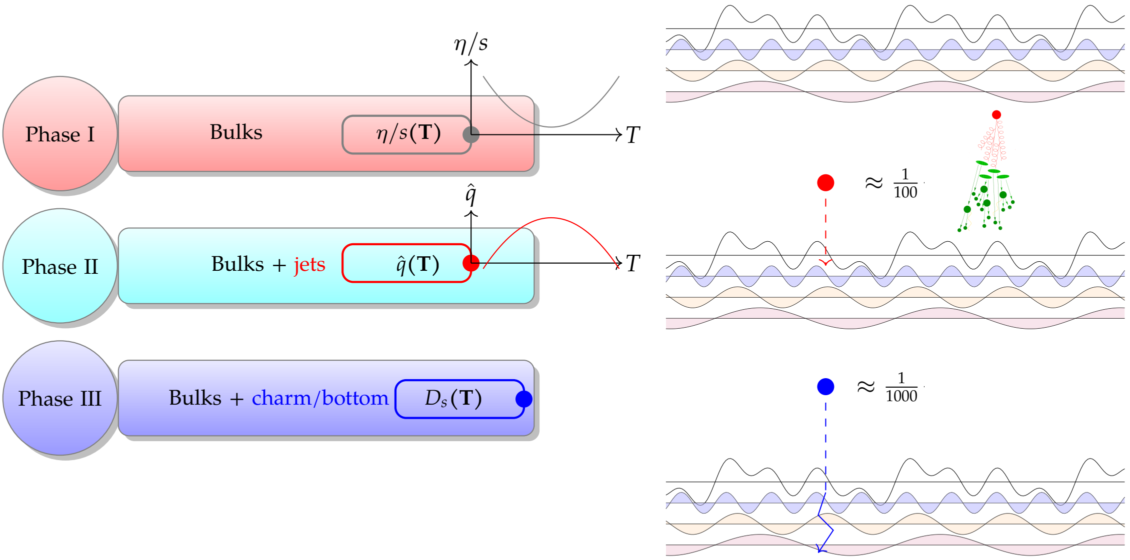
- CSC for providing the ∼24 million CPU hours
- Harri Niemi, Kari Eskola, Jonah E. Bernhard, J. Scott Moreland and Steffen A. Bass for their useful comments

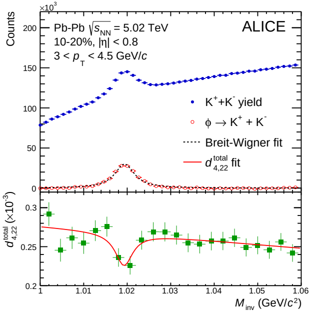
Transport properties



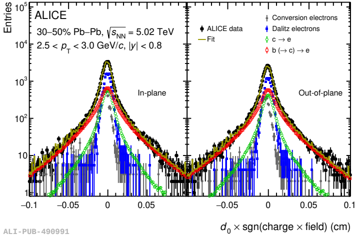
Heavy flavour obsrvables

$$
\begin{aligned}
\mathrm{SC}(m,n) \equiv {} & \langle\langle\cos(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)\rangle\rangle_c \\
= {} & \langle\langle\cos(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)\rangle\rangle \\
& -\langle\langle\cos[m(\varphi_1-\varphi_3)]\rangle\rangle\,\langle\langle\cos[n(\varphi_2-\varphi_4)]\rangle\rangle \\
= {} & \left\langle v_m^2 v_n^2\right\rangle - \left\langle v_m^2\right\rangle\left\langle v_n^2\right\rangle
\end{aligned}
$$

- Replace φ_1 or φ_2 with HF candidates
- Invariant mass or DCA approach
- compared to all tracks

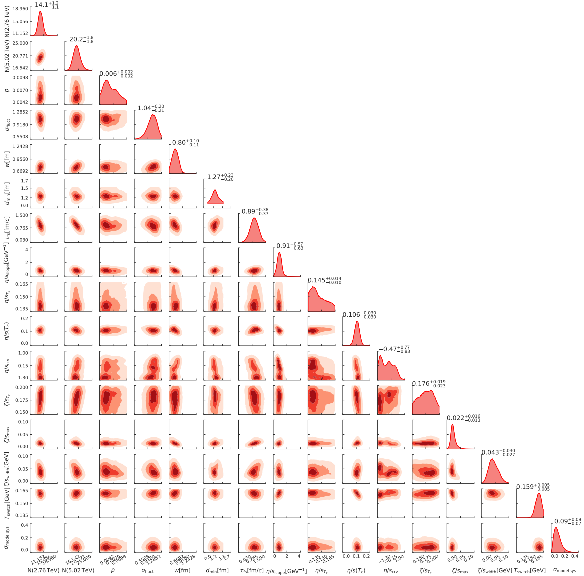


ALICE, JHEP06 (2020) 147

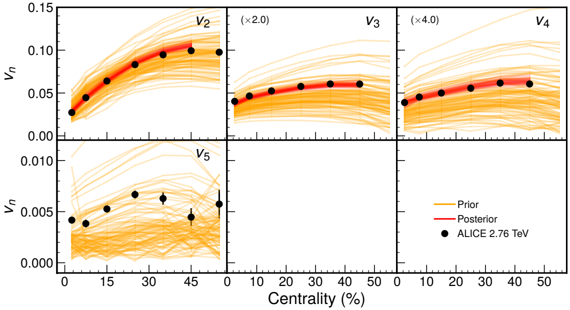


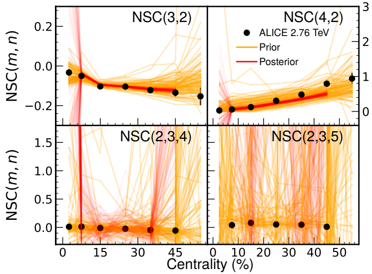
ALICE,Phys. Rev. Lett. 126 (2021) 162001

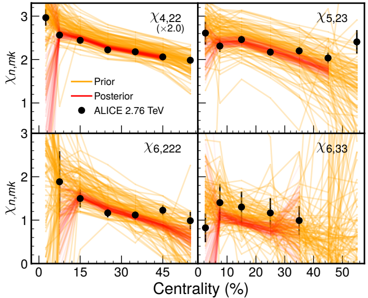
Marginal and joint marginal parts of the poster distribution



Design parametrizations







Concerns on the higher order harmonics